Impulse/Momentum, Force, and Energy Analyses
for a collision between
MICHIGAN/GREAT LAKES and LINDA E

This purpose of this analysis is to estimate the reactions of the Integrated Tug and Barge (ITB) MICHIGAN/GREAT LAKES and the LINDA E, had these vessels collided. This analysis is necessary to gain a better understanding of:

1. How and why the LINDA E sank.
2. How the damage profile found on the LINDA E was created.
3. How the marks found on the bow of the GREAT LAKES were created.
4. How the motions of the LINDA E immediately after impact acted to preserve the damage profile.
5. Whether or not the crew on the ITB would have easily detected the collision.

Predicting the motion of a vessel is by no means a trivial problem. This is true even when all the required information necessary to form the governing equations of motion is available. Added to this, the scenario proposed involves a very complicated interaction between two vessels.

In this case, most of the information necessary to perform this type of analysis is unavailable. Of the pieces of information missing, the most critical relate to forces and motions upon the LINDA E just prior to impact. There is no information regarding external forces due to wind or seas, throttle changes, rudder movements, etc. There is also no information on the linear or angular velocities and accelerations of the LINDA E prior to the incident. Therefore, I will not attempt a rigorous analysis to determine the exact motions or forces upon the vessels at the time of the collision.

Fortunately however, the collision scenario suggested by observed markings and damage lends itself to some reasonable, simplifying assumptions that allow me to estimate (or at least qualitatively analyze) reactions upon the vessels. This is particularly true on the ITB, which expectedly would experience significantly less change in motion than the fishing vessel. Thus, with considerable assistance from LT DeWane Ray, PE and others from the Coast Guard Marine Safety Center I have prepared the following:

I. Estimate of Speed Reduction of Integrated Tug and Barge (ITB) Due to Collision.
II. Qualitative Analysis of Forces and Vessel motions.
III. Qualitative Energy Analysis

Where necessary, assumptions were chosen so as to affect the ultimate answer in the most conservative way possible. For example, when estimating the maximum possible velocity change for the ITB, I assume a fully plastic, concentric collision, thereby ensuring largest affect possible.

These analyses are intended only as general indicators of the relative motions and reactions upon the two vessels. A more rigorous analysis would be necessary to determine the exact forces and motions of the two vessels.

LCDR Bryan R. Emond, PE

Impulse/Momentum, Force, and Energy Analyses
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I. Estimate of speed reduction of Integrated Tug and Barge (ITB) due to a collision:

A. Variables Defined:

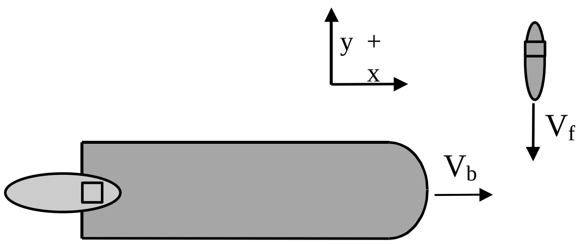


Figure (1)

V_b = Velocity of ITB
V_f = Velocity of fishing vessel
V’ = Velocity of both after collision
m_b = Mass of ITB
m_f = Mass of fishing vessel
L_f = Length of fishing vessel
T_f = Draft of fishing vessel
I_{bz} = mass moment inertia for ITB
ω_b = rotational velocity of ITB
ω' = rotational velocity of both after
r_1 = distance, ITB c.g. to impact pt
r_2 = distance, ITB c.g. to center of rotation

B. Assumptions:

1. Assuming a completely plastic, oblique central impact. Although this assumption is not realistic or consistent with the marks or damage observed, this will result in the largest possible reduction in speed for the ITB and is thus conservative for the purposes of this analysis.
2. Assuming that forces such as the propeller thrust and drag (resistance) on the ITB remain relatively constant and equalized during the very short (less than 1 second) collision period.
3. Neglecting the added drag force of pushing a fishing vessel transversely through water. While significant, this force is a function of velocity and would not be full developed until the vessel had achieved its maximum transverse speed. Additionally, during initial contact between the vessels, the location and direction of this drag force would act more to increase the heeling moment upon the fishing vessel than would it act as a resisting force upon the barge.
4. Assuming a nearly perpendicular collision, consistent with the damage observed.
5. Neglecting the “added mass” of the water travelling with the ITB. While this term is generally significant, neglecting this added mass will result in the largest possible reduction in speed for the ITB and is thus conservative for the purposes of this analysis.

C. Estimating the weights & masses of the two vessels:

1. Estimated displacement of the LINDA from MSC Stability Analysis ≈ 46,400 lbs. = 20.7 Long Tons (Ltons),
   ∴ weight of the fishing vessel (W_F) = **20.7 Ltons = 46,400 Lbs**.
2. Estimated displacement of ITB MICHIGAN/GREAT LAKES:

Dimension	**Tug MICHIGAN**	**Barge GREAT LAKES**
Length (L)	112.6’	414.1’
Beam (B)	27.1’	60.1’
Draft Aft (T_A)	19’	14’
Draft Fwd (T_F)	19’	13’
Mean Draft (T_M)	19’	13.5’
Est. Block Coef (C_b)	.59	.78

Displacement ≈ $\nabla = C_b LBT/36$ (36 for fresh water)

$\nabla_{Tug} \approx .59 \times 112.6 \times 27.1 \times 19.0 / 36 =$ **950 Ltons**
$\nabla_{Barge} \approx .78 \times 414.1 \times 60.1 \times 13.5 / 36 = 7280$ Ltons, or more accurately,
∇_{Barge}(from barge Trim & Stability Booklet) = **7250 Ltons**

Weight of ITB (W_{ITB}) = $\nabla_{Tug} + \nabla_{Barge}$ = **8200 Ltons = 18,368,000 Lbs**

Note: Based upon the above, the ITB has 396 times the mass of the fishing vessel.

D. Estimating the velocities of the two vessels:

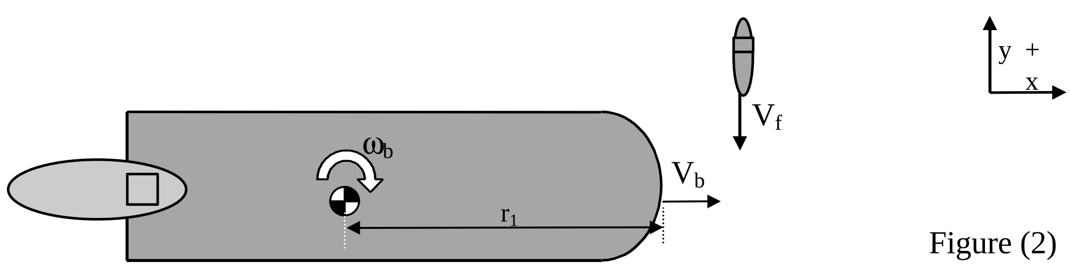


Figure (2)

1. Along the x-axis:
   $V_{bx} = V_b = 12$ Knots = 20.3 feet/second
   $V_{fx} = 0$
2. Along the y-axis:
   $V_{by} = 0$
   $V_{fy} = V_f = 8.5$ Knots = 14.3 feet/second (Estimated at max cruising speed)

E. Equating linear system momentum before and after collision:

1. $V_b m_b + V_f m_f = V'(m_b + m_f)$ **(1)**

2. $V' = \dfrac{(m_b V_b + m_f V_f)}{(m_b + m_f)}$

3. As W = mg and g = constant

4. $V' = \dfrac{(W_b V_b + W_f V_f)}{(W_b + W_f)}$

F. Considering only those velocities along the x-axis:

1. $V_x' = \dfrac{(W_b V_{bx} + W_f V_{fx})}{(W_b + W_f)}$

   $= \dfrac{(8200 \text{ Ltons})(12 \text{ Knots}) + (20.7 \text{ Ltons})(0 \text{ Knots})}{(8200 \text{ Ltons} + 20.7 \text{ Ltons})}$

   = **11.97 Knots**

2. This is a speed reduction of approximately **0.03 Knots (0.05 ft/s)**

G. Equating rotational system momentum about the z-axis before and after collision:

1. $I_{bz}\,\omega_b + m_f V_{fy} r_1 = (m_b + m_f) V' r_2 + I_{bz}\,\omega'$ **(1)**
2. Assuming no initial rotation of barge, $\omega_b = 0$
3. Assuming final rotation about center of gravity of barge, $r_2 = 0$
4. Solving for ω' this equation becomes:

   $$\omega' = m_f V_f r_1 / I_{bz}$$

5. Neglecting the mass moment of inertia for the tug and estimating the mass moment of inertia for the barge, I_{bz}, from the formula for a rectangular prism:

   $$I_{bz} = m_b (L^2 + B^2)/12 = 8.32 \times 10^9 \text{ lb-ft-s}^2$$

6. Estimating r_1 at approximately half the overall length of the ITB; r_1 = 227'.
7. Inserting these values into the equation for ω' above:

   $\omega' = 5.6 \times 10^{-4}$ radians/sec **= .03 degrees/sec** = 2 degrees/minute

H. Estimation of time over which deceleration occurred:

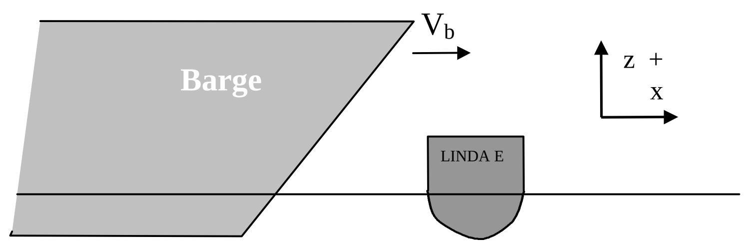


Figure (3)

1. The time period over which this collision occurred is not trivial and is dependent upon many factors which cannot be determined to a certainty. However, we can reasonably estimate this time if we assume that:

a. The impact forces only acted during the time that deformation to the LINDA E hull occurred, and
b. The LINDA E did not move transversely during this deformation period.

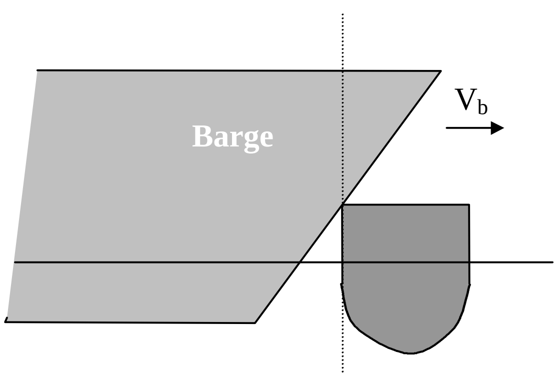


Figure (4): Initial Contact

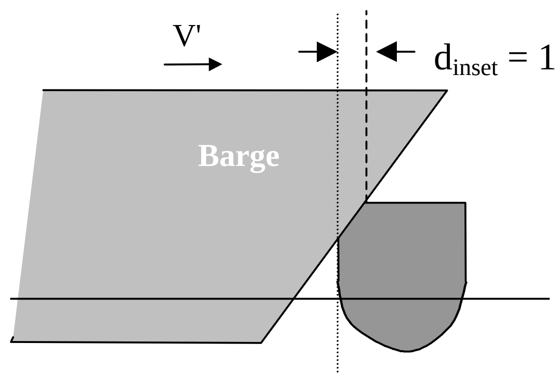


Figure (5): End of Impact

2. From the damage profile, we have observed that the deck edge of the LINDA E is inset approximately 1'. Assuming the LINDA E did not roll or slip sideways, at 12 Knots (20.3 feet/second) the impact would have lasted at least 1/20th of a second. See figures (4 and 5)

$\Delta t = d_{inset} / V'_x =$ **0.049 seconds**

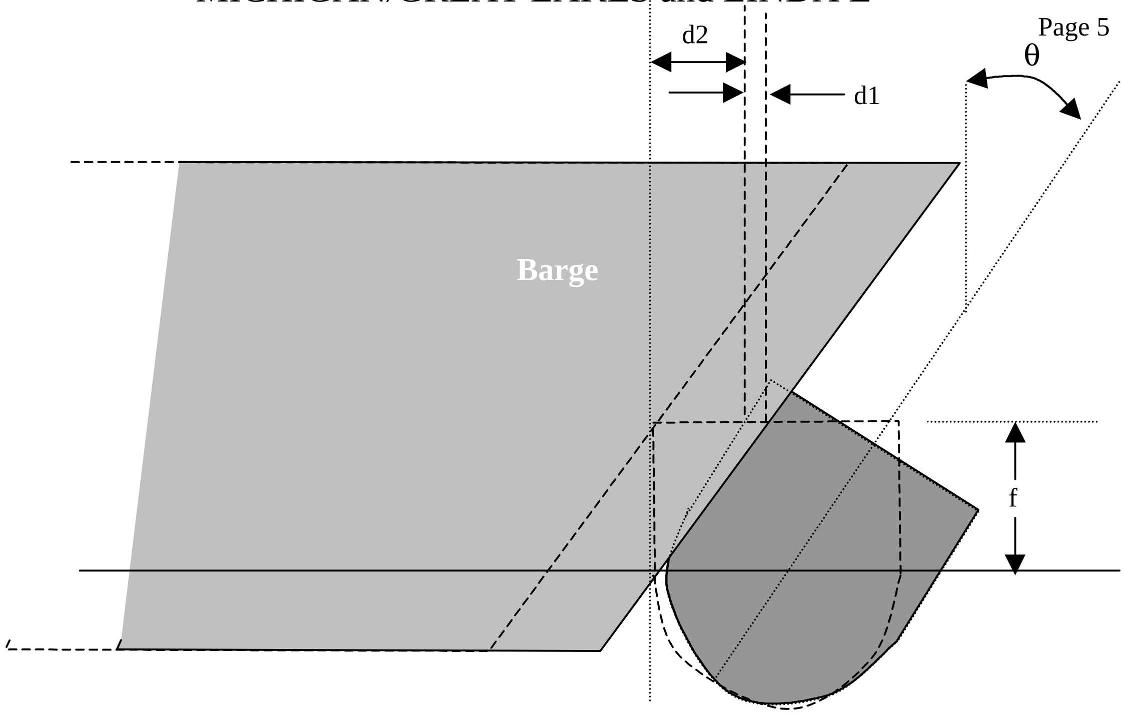


Figure (6)

3. If we consider that the LINDA E likely rolled during the collision, this extends the impact period an additional lateral distance as shown in Figure (6). Distance **d1** is the distance attributed to the inset, taking into account the angle of inclination. Distance **d2** is the distance added due to roll. The total of these distances can be estimated by knowing the freeboard (**f**) of the LINDA E and the maximum angle of roll (θ). Based upon the MSC's stability analysis and damage profile model, the LINDA E had a freeboard (**f**) of approximately **5' 11"** and rolled approximately $\mathbf{51^0}$.

$d1 = f \text{ Tan } \theta = 7.3'$

$d2 = d_{inset} / \text{Cos } \theta = 1.6'$

$d_{total} = d1 + d2 = 8.9'$

Which gives a time period for impact is equal to

$\Delta t = d_{total} / V'_x =$ **0.44 seconds**

J. Estimation of deceleration of the barge:

1. If we make the simplifying assumption that deceleration was constant over the extremely short period of impact (impulse), we can use the equation below to estimate the magnitude of deceleration.

$V'_x = V_{bx} + a\,(\Delta t)$

$a = (V'_x - V_{bx})/(\Delta t)$

2. Using the two Δt's developed above, we can bracket the magnitude of this acceleration:

for $\Delta t = 0.27$ seconds: $a = -0.1\ ft/s^2$

for $\Delta t = 0.049$ seconds: $a = -1.0\ ft/s^2$

3. Therefore the deceleration of the ITB upon impact was most likely between 0.1 ft/s^2 and 1.0 ft/s^2. Based upon the damage observed which indicates that the fishing vessel most likely rolled during impact, the deceleration was probably closer to 0.1 ft/s^2.

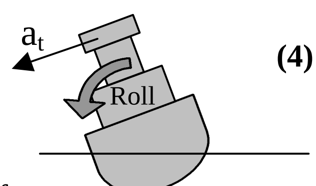
4. In comparison:

A. The minimum threshold for a person to detect a linear deceleration is 0.08 g's or 2.6 ft/s^2. **(2)**

B. Typical office elevators have accelerations and decelerations in the range of 2 ft/s^2 to 8 ft/s^2. **(3)**

C. Person's on a vessel are continuously subjected to some sort of acceleration, particularly in the transverse direction as the vessel rolls. The tangential acceleration (**a_t**) that would occur in the pilothouse of the MICHIGAN from rolling the vessel can be estimated using the equation below:

$a_t = r(4\pi^2/T_0^2)\phi$ **(4)**



where

ϕ = maximum roll angle in radians

r = radius from center of gravity of vessel in feet

T_0 = period of roll in seconds

Assuming reasonable values for each variable:

$\phi = 10^0$ roll $= 10\pi/180$

$T_0 = 15$ second roll period

r = 42' (height of upper pilothouse above ITB Center of gravity)

$a_t = 42(4\pi^2/15^2)(10\pi/180) = \mathbf{1.3\ ft/s^2}$

REFERENCES:

(1) Vector Mechanics for Engineers, Beer & Johnston, 5th Edition, Chapter 13
(2) Human Engineering Performance: A Guide for System Designers, Robert W. Bailey, Prentice Hall
(3) Method for Calculation of Elevator Evacuation Times, Klote, J. H, Journal of Fire Protection Engineering, Vol. 5, No. 3, 83-96, 1993
(4) Principles of Naval Architecture, SNAME publications, 1967, Chapter IX, Section 4, Edward V. Lewis

II. Qualitative analysis of forces and vessel motions.

A. Variables Defined (in addition to those defined in Section I):

F_T = Thrust Forces from Propulsion
F_R = Rudder Force
F_D = Drag forces on hull (wave, skin, etc.)
F_W = Vessel weight or displacement, assumed as acting through center of gravity: ◕
F_B = Buoyant force
F_I = Contact Force between two vessels
θ = Angle of Heel
M_R = Righting moment = $f(F_{b,} \theta)$
M_T = Trimming moment = $f(F_{b,}$ Trim)

Additional Subscripts:
b - Integrated Tug and Barge
f - Fishing Vessels

B. Assumptions:

1. Assuming Integrated Tug and Barge (ITB) act as one rigid body.
2. Assuming pre-impact forces on ITB, including propeller thrust (F_T), drag (F_D), etc. are equalized and remain relatively constant throughout very short period of impact.
3. Assuming initial ITB velocity as relatively constant, and therefore surge is negligible.
4. Neglecting all pre-impact rotational forces and motions for ITB (roll, pitch, and yaw) as sea conditions on date of this incident were relatively calm and the vessel was travelling a relatively straight course. Also considering ITB rudder forces as negligible to impact.
5. Considering all pre-impact transverse and vertical accelerations (sway and heave) as negligible for the same reasons as stated in assumption 4.
6. Assuming a nearly perpendicular collision, consistent with the damage profile.

<u>C. Interaction between the two vessels</u>:

1. Based upon the damage profile and marks found on the barge, the two vessels most likely initial point of contact between the two vessels would have been the stem of the barge GREAT LAKES and the upper deck edge of the LINDA E. See Figure (7) and (8).

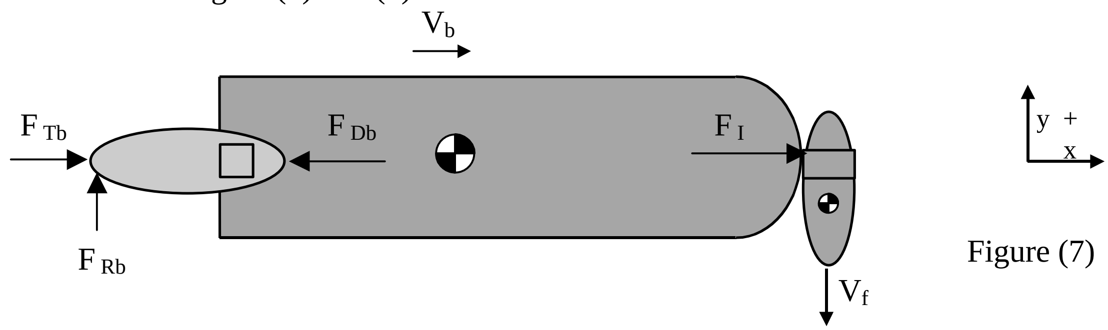


Figure (7)

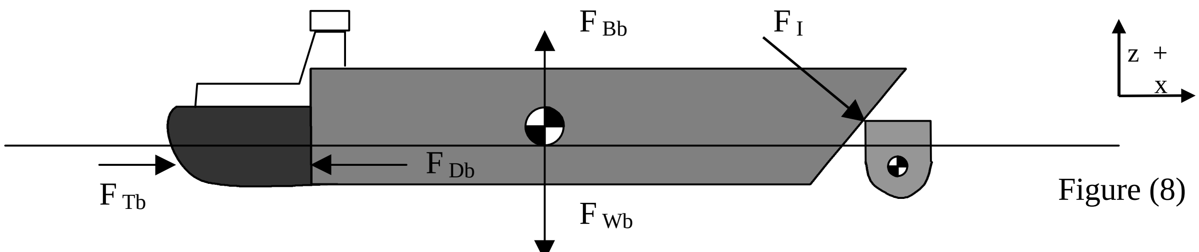


Figure (8)

1. The contact force between these two vessel, F_I, though likely not constant throughout the collision, would have to be equal and opposite throughout the period of contact. The angle ϕ through which this force would act is dependent upon a number of unknown variables and therefore is not determinable. However, we may use this angle to express the components of this force in terms of our reference frame. See Figure (9).

$$F_{Ix} = F_I \text{ Cos } \phi$$

$$F_{Iz} = F_I \text{ Sin } \phi$$

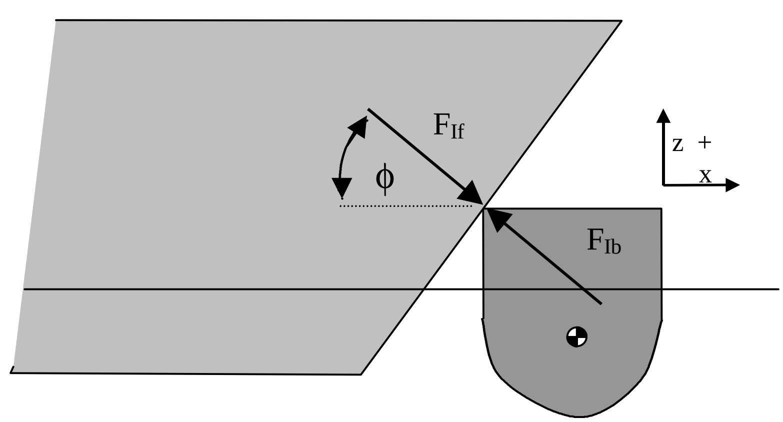


Figure (9)

2. The impact force upon the fishing vessel, F_{If} is the result of many components:
   - F_{Impact}- initial impact force
   - F_{Push} - steady state force required to push the fishing vessel transversely
   - F_{BW} - downward force imposed by the weight of the barge.
   - $F_{Friction}$ - force from barge stem sliding on hull of fishing vessel

The magnitude and direction of these components vary with heel angle and/or time as shown in the freebody diagram:

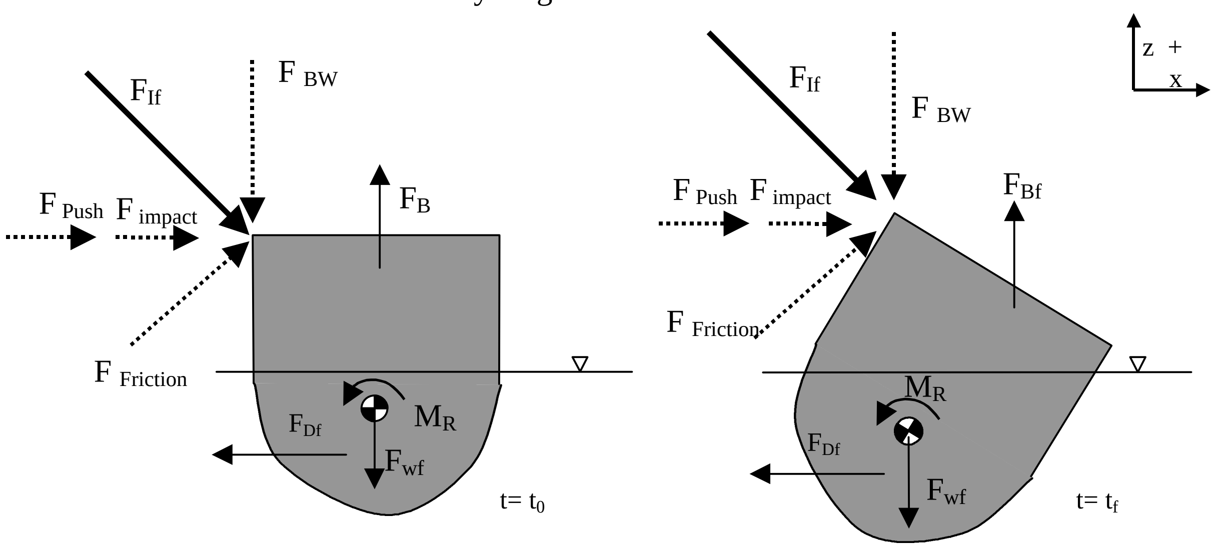


Figure (10)

Looking at the characteristics of the individual forces:

a. F_{impact} - This force acts primarily in the positive x-direction. The magnitude of this force is dependent upon the impulse provided by an 8200 Lton ITB travelling at 12 Knots. In general, we would expect F_{impact} to be large, but very short-lived.

b. F_{Push} - This force acts primarily in the positive x-direction. This is a reaction force to:

(1) Acceleration of the mass of the fishing vessel. Initially, the fishing vessel's acceleration would be high but then reduce as the speed of the fishing vessel approaches the speed of the barge. The force required to create this acceleration would increase and decrease accordingly.

(2) Drag of pushing a fishing vessel sideways though the water. The drag force, F_{Df}, being primarily a function of velocity, would not be fully

developed unit the fishing vessel reached its maximum transverse speed.

Therefore, we expect that force F_{Push} would be high initially, might reduce somewhat, and then increase rapidly until the fishing vessel reached its maximum transverse velocity or rotated off the bow of the barge.

c. F_{BW} - This force acts primarily in the negative z-direction. It is readily apparent that the barge has sufficient weight to sink the LINDA E. However, the magnitude of the force F_{BW} is dependent upon the upward reaction force provided by the LINDA E. This reaction force is dependent upon two variables: (1) amount and direction of heeling angle (which results in a righting moment) and (2) parallel sinkage (how much the LINDA E is pushed downward by the barge), and change in trim of the LINDA E.

Since the damage profile suggest that the LINDA E heeled away from the bow of the barge and not towards it, the expected righting moment would have acted in the wrong direction to have contributed to F_{BW}.

As we do not know the extent that the barge might have pushed the LINDA E down into the water, the contribution of parallel sinkage is not certain. The reserve buoyancy from the fishing vessel limits the upper bound of this force. MSC's stability analysis estimated the reserve buoyancy of the LINDA E at 30.2 Ltons. However this value assumes no heel and no downflooding. As the vessel heels, the downflooding point (for example the open port service door) moves closer to the water and the magnitude of reserve buoyancy is reduced. From the MSC's stability analysis, downflooding begins when the vessel heels 22.7^0 to port. (See Figure 13)

The contribution of the change in trim is also not certain. The trimming moment, acting around the x-axis would act to increase this force, but only until downflooding occurred. However, combined with the affects of heel described above, the expected trim aft would mostly serve to hasten downflooding and reduce reserve buoyancy.

While the damage profile suggests some initial downward force, the actual magnitude of F_{BW} would most likely have been considerably less than 30.2 Ltons. In general, we would expect this force to be relatively small initially, grow gradually until downflooding and then reduce rapidly.

d. $F_{friction}$ - This force acts along the contact surface in the positive x and z direction. The magnitude of this force is dependent upon the force normal to the contact surface and the coefficient of friction between the two vessels. The magnitude of the force normal to the contact surface is in turn dependent upon the $F_{Impact,}$ $F_{Push,}$ F_{BW}, and the direction these forces act through. Accordingly, we would expect this force to start out fairly large and remain significant until the LINDA E rotated off the bow or sank.

Combining components of F_I along the x and z- axes:

$$F_{Ix} = F_{Impact} + F_{Push} + F_{Friction} \sin \phi$$

$$F_{Iz} = -F_{BW} + F_{Friction} \cos \phi$$

3. Because of the number of unknown and independent variables, it is not possible to determine the exact magnitude or direction of $F_{If.}$ However qualitatively, we see that initially, the most significant components of this force tend to be along the x-axis. The components along the z-axis may have combined to act upward or downward.

D. Reaction on the Fishing Vessel:

1. **Heeling Moment:** As previously described, the general indication is that initially, the contact force upon the LINDA E (F_{If}) acted in a direction more along the positive x-axis (and may have even acted upward). As this force acted above the vessel's center of gravity, the moment generated by this force would have heeled the vessel away from the bow of the vessel. See Figure (11). As shown in figure (12), the transverse drag (F_{Df}) force would act through the centroid of the underwater profile. Because this is below the vessel's center of gravity, F_{Df} would act to increase the heeling moment. The massive heeling moment developed would most likely overcome the rotational inertia and righting moment (M_R) of the fishing vessel, at least to the point that downflooding would occur. See figure (13). This heeling motion is also consistent with the damage found on the fishing vessel.

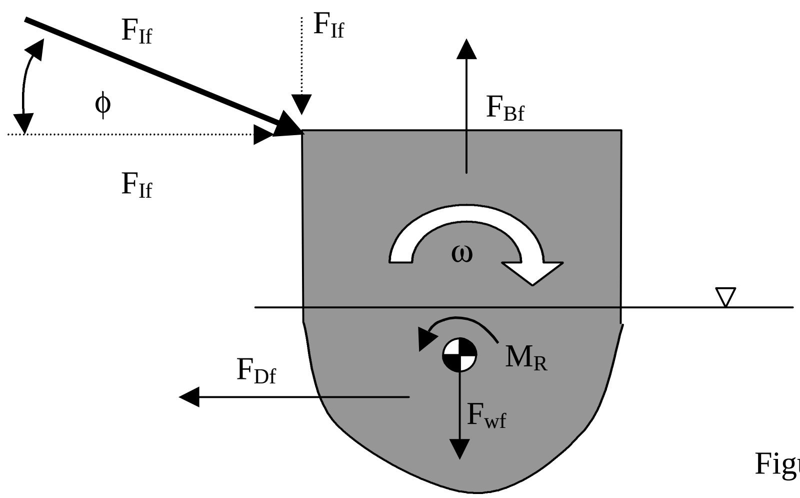


Figure (11)

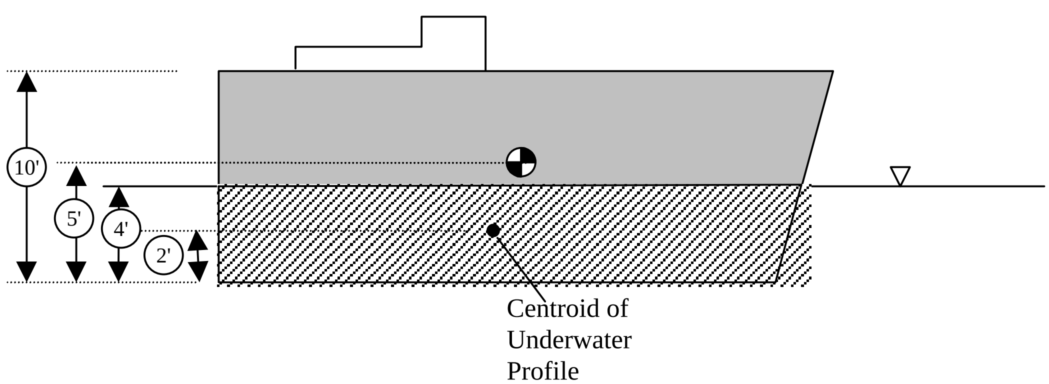


Figure (12)

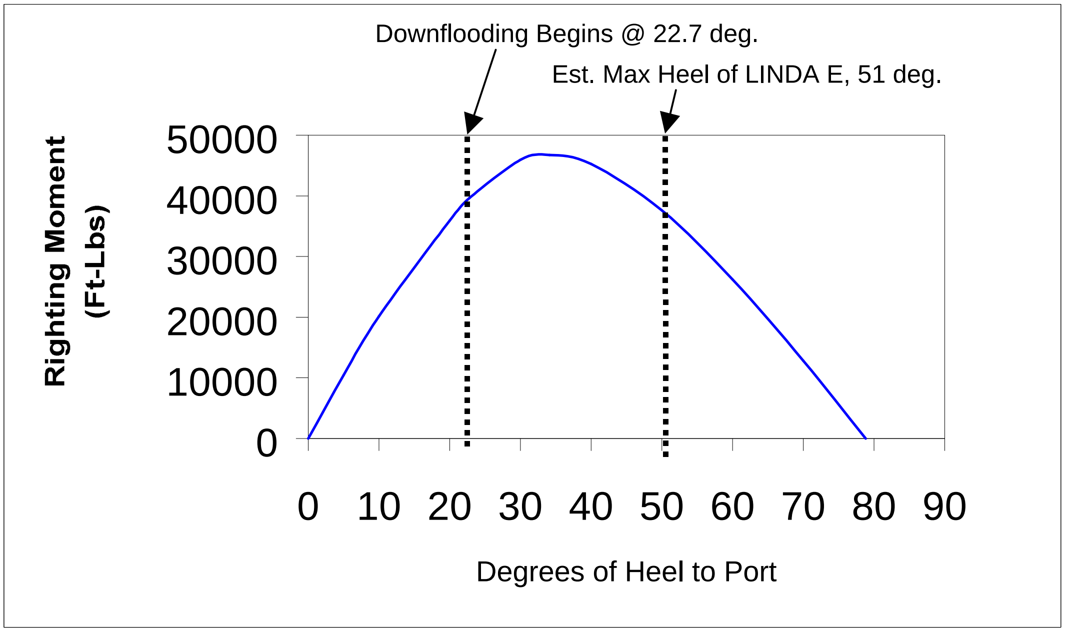


Figure (13)

2. **Yawing Moment**: The force F_{If} acted through a point well aft of the fishing vessels longitudinal center of gravity. As there were likely no forces substantial enough to resist this moment, the vessel would have yawed to it's starboard. Markings found on the vessel and barge are consistent with this kind of rotation.

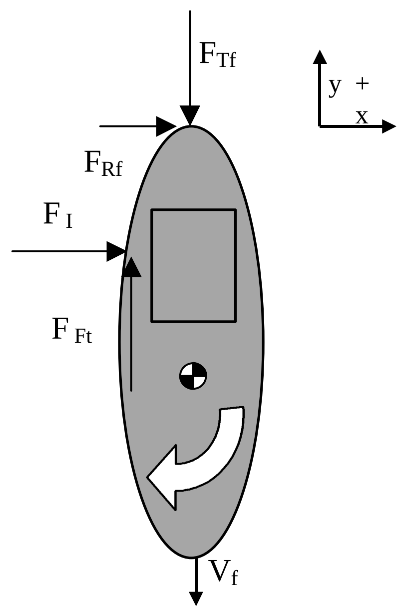


Figure (14)

3. **Trimming Moment**: may have acted vertically somewhat. As the force F_{If} acted through a point well aft of the fishing vessels longitudinal center of floatation, this force may have caused the vessel to trim. However, the direction of this force or the affect it might have had is not clear.

E. Reaction on the ITB:

1. **Heeling Moments**: As F_{Ib} acted through the transverse and longitudinal center of gravity of the ITB, no moments were generated about the x-axis. See Figure (7).

2. **Yawing Moments**: If the fishing vessel were moving before the collision, there may have been some moment generated about the z-axis of the ITB. The only force that would have been transferred in this direction would have been a transverse frictional force (F_{Ft}) between the stem of the barge and side of the fishing vessel. See figure (14). As shown in part I, section G of this analysis, even if we assume solid contact at the fishing vessel's maximum cruise speed, the resulting yaw is negligible.

3. **Trimming Moments:** At drafts of 13 feet Forward, 14 feet aft, the barge has a Moment to Trim One Inch ($M_{T1"}$) of approximately 1275 ft-Ltons. The largest vertical force possible from the fishing vessel would be from its reserve buoyancy of 30.2 Ltons. This force would act at the stem, approximately 190 feet from the Longitudinal Center of Floatation (LCF). Theoretically, this could result in a change in trim of 4.5 inches. However, as it appears that the fishing vessel heeled away from the barge downflooded very quickly, the amount of upward force and any resulting change in trim was probably much less. The downward component of the frictional force ($F_{friction}$) on the barge (complementary to that on the LINDA E) would also lessen the resulting trim.

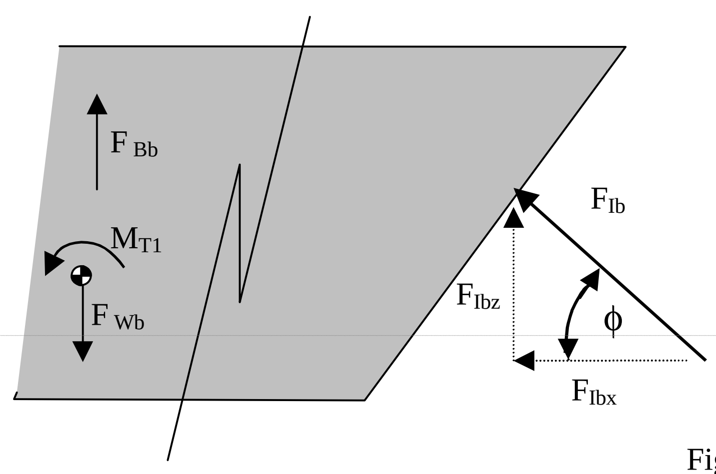


Figure (15)

III. Qualitative analysis of energy transformations.

A. There is insufficient information available to quantitatively determine the exact forces, moments, velocities and accelerations that would have been developed on the LINDA E from a collision with the GREAT LAKES. By the law of conservation of energy, however, we know that energy of the system before the impact must be equal to the energy after, less any energy dissipated from the system. Considering each form of energy qualitatively we find energy from the impact would have been transformed into a number of forms:

1. **Kinetic energy** in the form of translation, yaw, heel and pitch:

   As described in section II, after the collision the LINDA E would have yawed to starboard, heeled to port, and to a lesser extent, pitched (or trimmed) by the stern. This is consistent with the markings on both vessels. The LINDA E would also have translated sideways somewhat. Given the short time period in which this motion would have occurred, this rotation would have been very rapid, requiring significant kinetic energy.

2. **Dissipation energy** in the form of hull drag and structural deformation:

   The magnitude of the drag force from pushing the LINDA E transversely through the water was possibly very large. This drag likely assisted the heeling and rotation of the LINDA E, and resulted in the dissipation of a significant amount of energy.

   The deformation observed on the LINDA E, while severe, is relatively minor compared to the forces that were available from a collision with such a large vessel. This indicates that the collision was not a very plastic impact and that relatively little of the available energy was dissipated through structural deformation.

3. **Potential energy** needed to overcome the vessel's righting and trimming moments:
   As shown in section II, many of the most significant forces involved in the impact acted to heel the LINDA E. Although some energy would have been expended in this manner, the LINDA E most likely provided relatively little resistance and heeled immediately upon impact. Once downflooding of the LINDA E began, this heeling resistance (righting moment) would decrease rapidly.

B. Section I of the impact/momentum analysis used the conservative assumption that this was a fully plastic collision. The findings above indicate that energy from an impact would be transformed into a number of forms. Relatively little of the available energy is revealed in the structural deformation observed on the LINDA E. While severe, this deformation is relatively minor compared to the forces that were available from a collision with such a large vessel, indicating a less than fully plastic collision. Therefore the linear velocity change experienced by the ITB would be even less than estimated by the impact/momentum analysis.

C. Section I of the impact/momentum analysis uses the conservative assumption that this was a fully concentric collision. Because the center of gravity of the LINDA E was forward and below the initial point of impact, the collision was actually eccentric. Therefore the linear velocity change on the ITB would be even less than estimated by the impact/momentum analysis.